2010 Marywood Mathematics Contest

Level I

Sponsored by

SEMI-GROUP

The Student Mathematics Club of

Marywood University

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Directions:

1. This exam consists of 40 questions on 6 pages. Please check to make sure that you have all the pages.

2. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess. Your score will be the number of correct responses.

3. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark only one answer for each question. Only those answers on the answer sheet will be counted.

4. There is a sheet of blank paper on the last page which you can tear off and use as scratch paper. You may also use the back of the pages.

5. NOTE: In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.

Please do NOT open the test until you are told to do so.
1. Compute \(7 + (9)(0^4) - 6 \cdot \left(\frac{25}{-2 + 7}\right)^0\).

A. 1 B. 7 C. 10 D. \(-26\) E. None of these.

2. How many pairs of parallel edges are there in the following octohedron?

A. 8 B. 6 C. 4 D. 2 E. None of these.

3. Solve the equation \(9x + 2(7x + 5) = 0\) for \(x\).

A. \(x = 12/5\) B. \(x = -10/23\) C. \(x = -5/23\) D. \(x = 15/6\) E. None of these.

4. Find the number which is one fourth of the way from \(\frac{5}{8}\) to \(\frac{3}{2}\) on the number line.

A. \(\frac{27}{32}\) B. \(\frac{22}{8}\) C. \(\frac{27}{31}\) D. \(\frac{21}{8}\) E. None of these.

5. If \(x\) and \(y\) are non-negative real numbers, then \((125x^{13}y^{14})^{1/3} = \)

A. \(5x^{13/3}y^{14/3}\) B. \(125x^{13/3}y^{14/3}\) C. \(5xy\)
D. \(25x^{13/3}y^{10/3}\) E. None of these.

6. Find the equation of the line that has the \(x\) and \(y\) – intercepts at \((-10, 0)\) and \((0, 8)\).

A. \(9x - 11y = -99\) B. \(7x - 11y = -77\) C. \(-8x - 9y = 72\)
D. \(7x - 9y = -63\) E. None of these.

7. Suppose that an operation \(\oplus\) is defined as \(a \oplus b = a^3 - b^2\). What is the value of \(2 \oplus (3 \oplus 4)\)?

A. \(-113\) B. \(-15\) C. 11 D. \(-120\) E. None of these.
8. If the radius of a circle is doubled, the ratio between its area (measured in cm$^2$) and circumference (measured in cm) will

A. remain the same.  
B. also double.  
C. triple.  
D. quadruple.  
E. None of these.

9. Two values of the linear function $f(x)$ are $f(3) = -5$ and $f(6) = 8$. If $f(x) = mx + b$, what is $m - b$?

A. $\frac{7}{39}$  
B. $\frac{213}{39}$  
C. $\frac{67}{3}$  
D. $\frac{-51}{13}$  
E. None of these.

10. A sale at a department store says that customers will receive 40% off their total purchase. Lucy buys three items: a coat regularly priced at $40, a pair of shoes at $25, and a purse at $15. What will her final bill be?

A. $32$  
B. $112$  
C. $48$  
D. $54$  
E. None of these.

11. In a league of seven teams, each team plays each of the other teams twice during the regular season. The top three teams make the playoffs. The playoffs are single elimination, and the top team receives a bye for the first round (i.e., the top team skips the first round and goes to the next round directly). How many games are played altogether?

A. 21  
B. 23  
C. 42  
D. 44  
E. None of these.

12. Find the quotient $Q(x)$ and remainder $R$ after simplifying $\frac{18x^2 + 39x - 70}{-3x - 6}$.

A. $Q(x) = -6x - 1, R = -76$  
B. $Q(x) = -6x - 1, R = -64$  
C. $Q(x) = 6x - 1, R = -76$  
D. $Q(x) = 6x - 1, R = -64$  
E. None of these.

13. The two legs $AB$ and $BC$ of a right triangle are in the ratio of $1 : 3$, what is the ratio $AD : DC$?

A. $1 : 3$  
B. $1 : \sqrt{10}$  
C. $3 : 10$  
D. $1 : 9$  
E. None of these.
14. How many real solutions does the equation $x^3 - x^2 + x - 1 = 0$ have?
   A. 0          B. 1          C. 2          D. 3          E. None of these.

15. If the two real roots of the equation $x^2 + 6x - 4 = 0$ are $x_1$ and $x_2$ with $x_1 > x_2$, what is $x_1 - x_2$?
   A. $-6$       B. $2\sqrt{13}$       C. 6       D. 2       E. None of these.

16. Gabby and Jen can plant a garden together in 4 days, but Gabby can do it alone in 6 days. How long will it take Jen alone to finish the job after Gabby works on it for 4 days by herself?
   A. 1 day       B. 2 days       C. 3 days       D. 4 days       E. None of these.

17. Two planes leave from the same airport at 8AM. One plane is flying north at 600mph, while the other flies west at the same speed. Approximately how far apart are the planes from one another at noon?
   A. 1700mi       B. 2400mi       C. 3400mi       D. 4800mi       E. 7200mi

18. Alfred purchased a combination of candy bars and bags of chips from the grocery store. Each candy bar costs $0.70, and a bag of chips costs $0.50. If he bought a total of 7 items and spent $4.50, how much of the $4.50 was spent on candy bars?
   A. $1.40       B. $2.10       C. $2.50       D. $2.80       E. None of these.

19. What is the $y$-intercept of the line $5x + 3y = 1$?
   A. $1/3$       B. 3       C. $1/5$       D. 5       E. None of these.

20. $\ln \sqrt{e^3} + 2 \ln e^{1/4} =$
   A. $7/2$       B. $\ln(e^{3/2} + e^{1/2})$       C. 2       D. $\sqrt{3} + 1/2$       E. None of these.

21. There are 2 dimes, 3 nickels, and 5 quarters in a bag. If two coins are selected from the bag at random, what is the probability of picking one quarter and one nickel?
   A. $2/9$       B. $2/15$       C. $1/3$       D. $2/3$       E. None of these.
22. What is the next number in the sequence 6, 9, 14, 21, 30, ...

   A. 43  B. 39  C. 47  D. 41  E. None of these.

23. You and your crew, all members of the local math club, the Sand-Reckoners, are walking down Drinker Street when you come upon your rival mathletes from the high school across town, the Calculators! You decide to throwdown on each other with weapons of math destruction. Your group’s three members have three Casio’s, two abacuses and four TI-89’s. The Calculators have two HP’s, four Sharp’s, three slide rules and one set of Napier’s bones. What is the ratio of superior (electronic) weapons to inferior (non-electronic) weapons?

   A. 9:10  B. 6:5  C. 8:3  D. 13:6  E. None of these.

24. Corey could save 30% off the original price on any one CD with a coupon. He has another coupon for 20% off the total purchase price in one transaction regardless of the number of items, but the two coupons cannot be used together in one transaction. He plans to buy two CDs, one priced at $18 and the other at $12. What is the absolute lowest amount that he must spend to buy these two CDs?

   A. $24.60  B. $26.40  C. $22.20  D. $19.68  E. None of these.

25. Given the line \( y = \frac{7}{2}x + 1729 \). What is the slope of a line perpendicular to the given line?

   A. \( \frac{7}{2} \)  B. \( \frac{2}{7} \)  C. \( -\frac{7}{2} \)  D. \( -\frac{2}{7} \)  E. None of these.

26. Once upon a time, three brothers inherited 750 gold coins from their rich father. The will states that the youngest should receive a certain number of these gold coins, the second youngest should receive 90% of what the youngest brother receives and the oldest son should receive \( \frac{2}{3} \) of what the second youngest son receives. How many gold coins would the oldest son receive?

   A. 270  B. 300  C. 200  D. 180  E. None of these.

27. Write an equation in slope-intercept form for the line that passes through the points (4,5) and (20,25).

   A. \( y = x + 1 \)  B. \( y = x + 5 \)  C. \( y = \frac{4}{5}x \)  D. \( y = 4x \)  E. None of these.
28. A right triangle has a base of 8 and a hypotenuse of 10. What is its area?
   A. 40  B. 24  C. 48  D. 12  E. None of these.

29. Expand and simplify the expression: \((a + b)^3 + (a - b)^3\).
   A. \(2a^3\)  B. \(2a^3 - 2b^3\)  C. \(2a^3 - 6a^2b + 6ab^2 - 2b^3\)
   D. \(2a^3 - 6ab^2\)  E. None of these.

30. What is the radius of the circle \(x^2 + y^2 + 2x + 4y = 2\)?
   A. 7  B. 5  C. \(\sqrt{7}\)  D. 2  E. \(\sqrt{5}\)

31. Which of the following points is on the line \(y = 5x + 4\)?
   A. \((24, 4)\)  B. \((4, 0)\)  C. \((14, 2)\)  D. \((2, -2/5)\)  E. None of these.

32. The circle \(x^2 + y^2 = 1\) and the parabola \(y = x^2 + k\) are tangent to each other at exactly two points. What is the value of \(k\)?
   A. 1  B. 4/5  C. -4/5  D. -5/4  E. None of these.

33. We define a two digit integer as “well-behaved” if it is equal to four times the sum of the two digits. What is the sum of all the two digit well-behaved integers?
   A. 120  B. 84  C. 72  D. 60  E. None of these.

34. Notice that \(2010 = 2 \times 3 \times 5 \times 67\) and 67 is a prime number, what is the number of ALL positive integer factors of 2010?
   A. 4  B. 8  C. 14  D. 16  E. None of these.

35. On a quiz with 4 multiple choice questions, each question has exactly five answers, where one and only one of these are correct. A completely clueless student randomly chooses one answer for each problem. What is the probability that this student will get at most one of the correct answers?
   A. \(\left(\frac{4}{5}\right)^3\)  B. \(\left(\frac{4}{5}\right)^4\)  C. \(2 \times \left(\frac{4}{5}\right)^3\)  D. \(2 \times \left(\frac{4}{5}\right)^4\)  E. None of these.
36. If \( x + y + z = 0 \) and \( x + y + z^2 = 6 \), what is the largest possible value of \( x + y \)?

A. 3  
B. 2  
C. −2  
D. −3  
E. None of these.

37. If \( m \) and \( n \) are both positive integers and \( m > 1 \), such that \( m | (28n + 25) \) and \( m | (7n + 3) \). What is \( m \)?

A. 9  
B. 11  
C. 13  
D. 17  
E. None of these.

38. The last digit of \( 13^{2010} \) is

A. 1  
B. 3  
C. 7  
D. 9  
E. None of these.

39. In the regular hexagon \( ABCDEF \), \( BF = 2\sqrt{12} \), what is the area of the hexagon \( ABCDEF \)?

A. 10  
B. 12  
C. 6  
D. \( 6\sqrt{12} \)  
E. None of these.

40. A bouncy ball is dropped to a flat surface from 100 meters above and each time it bounces back up, it goes to half of its previous height, in other words, the first bounce sends it back up to 50 meters above the surface and the next bounce sends it back up to 25 meters, etc. What is the total distance (in meters) traveled by the bouncy ball if it is stopped exactly when it reaches the surface before the 10th bounce? (For the purpose of this problem, ignore the size of the ball.)

A. \( 100 \times \frac{6147}{2048} \)  
B. \( 100 \times \frac{3068}{1024} \)  
C. \( 200 \times \frac{1023}{1024} \)  
D. \( 400 \times \frac{1023}{1024} \)  
E. None of these.
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